New Search Strategies for the Petri Net CEGAR Approach

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Outline of the talk

1. Introduction
2. The CEGAR approach on Petri nets
3. New iteration strategy
4. Search strategies
5. Evaluation
6. Conclusions

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Introduction – Reachability analysis

- Reachability analysis
  - Is a given marking reachable from the initial marking?
  - Drawback: complexity
    - Decidable [Mayr’81]
    - At least EXPSPACE-hard [Lipton’76]
    - No precise upper bound is known
  - Possible solutions
    - Partial order reduction
    - Symbolic methods
    - Abstraction

Introduction – Abstraction

- Abstraction
  - General approach to handle large (infinite) state spaces
    - Less detailed (finite, smaller) state space representation
  - Abstraction refinement is required
    - A behavior in the abstract model may not be realizable
    - Refine using information from the explored part
  - → **C**ounter**E**xample **G**uided **A**bstraction **R**efinement
  - Applying CEGAR on Petri nets [Wimmel & Wolf'11]

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CEGAR approach on Petri nets

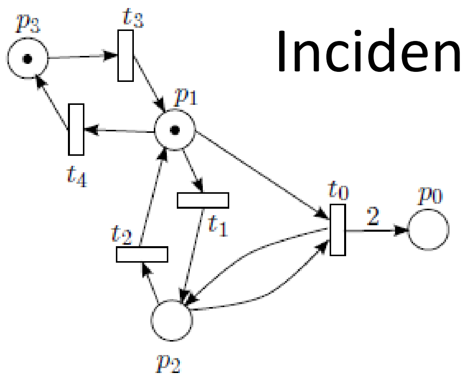
- Abstraction of Petri nets: state equation

Initial marking

$$m_0 + Cx = m_1$$

Target marking

Incidence matrix



$$\begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ -1 & -1 & 1 & 1 & -1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

Firing count of transitions *(unknown)*

Reachability problem

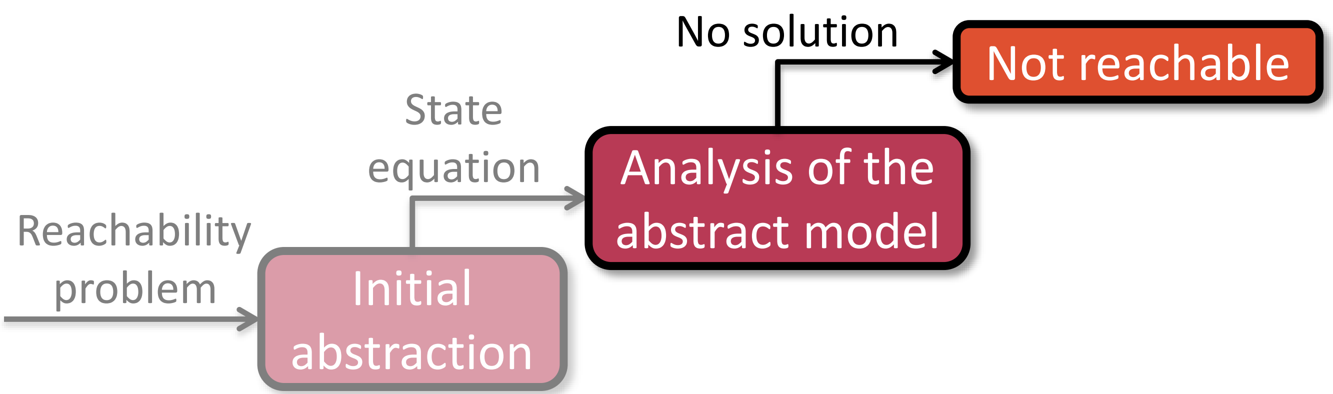
Initial abstraction

State equation

CEGAR approach on Petri nets

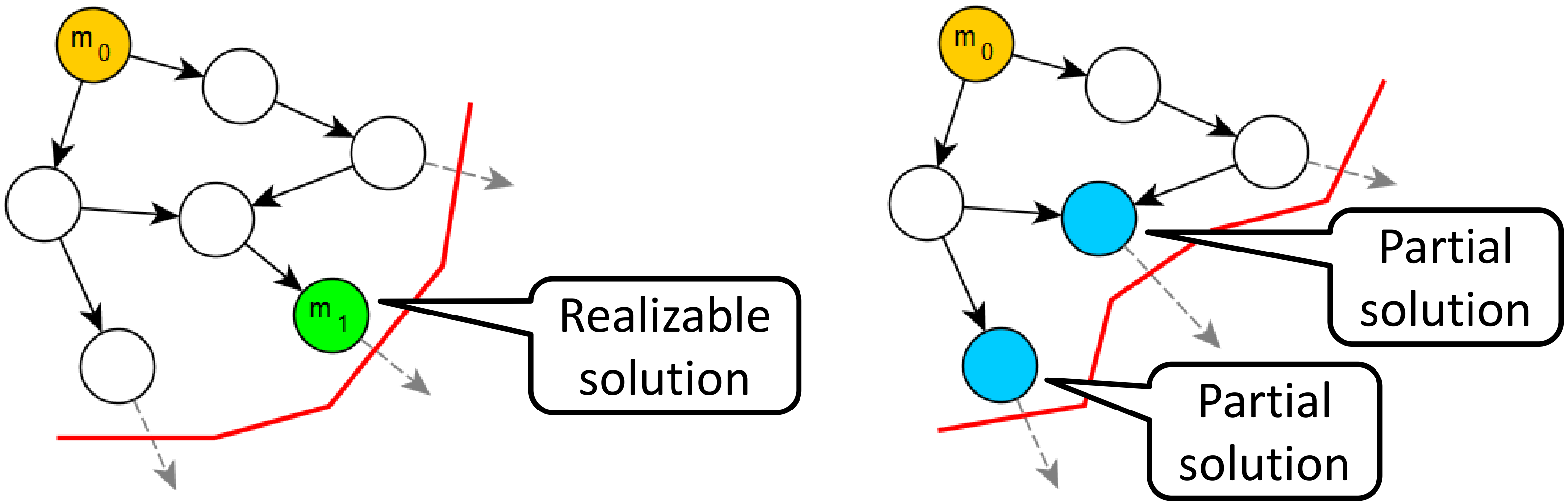
- Analysis of the abstract model
  - Solving the state equation for the firing count of transitions
  - Integer Linear Programming problem
  - Necessary, but not sufficient criterion for reachability

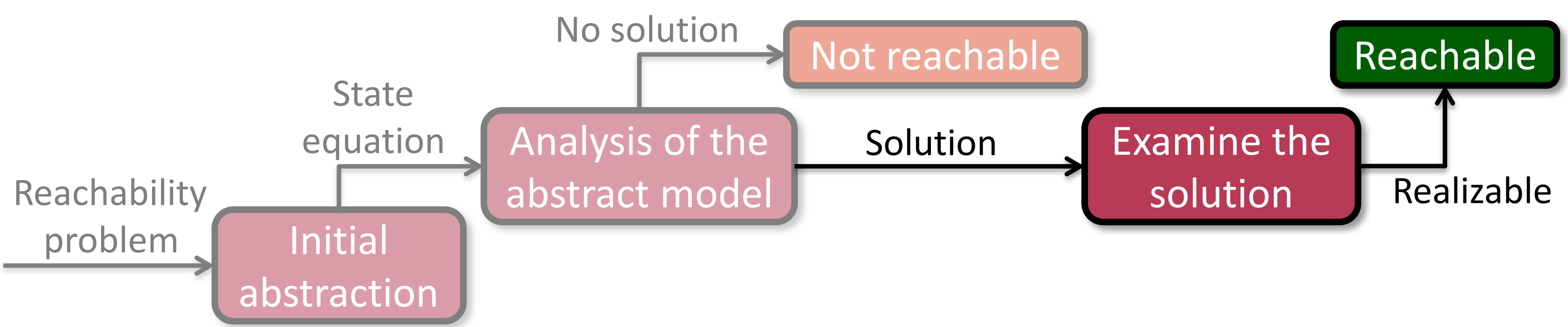
$$m_0 + C\mathbf{x} = m_1$$



CEGAR approach on Petri nets

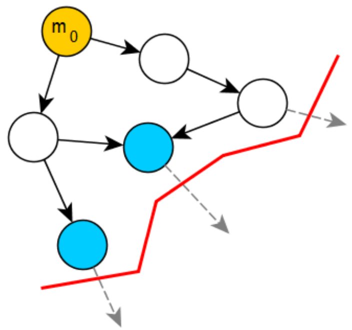
- Examining the solution
  - Bounded exploration of the state space

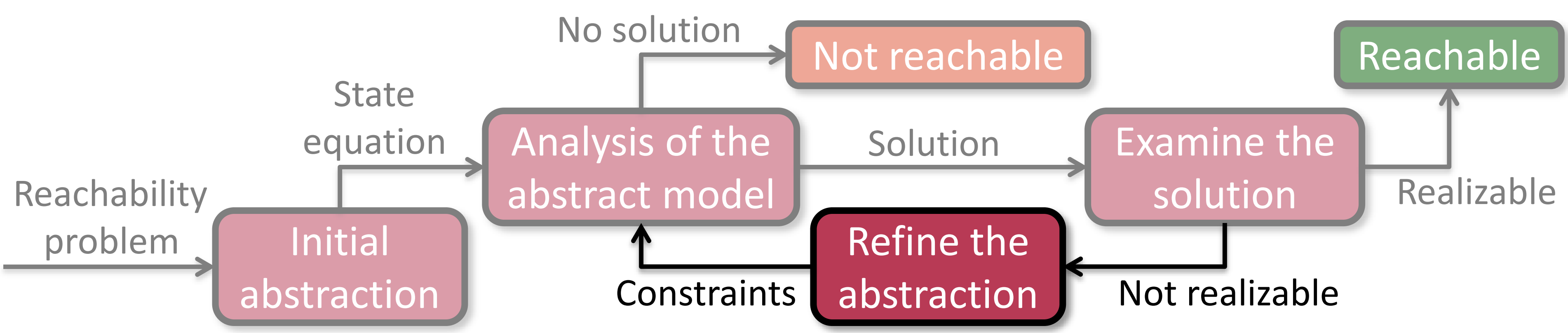




CEGAR approach on Petri nets

- Abstraction refinement
  - Exclude the counterexample without losing any realizable solution
  - Constraints can be added to the state equation
    - The state equation may become infeasible
    - A new solution can be obtained
  - Traversing the solution space of the state equation

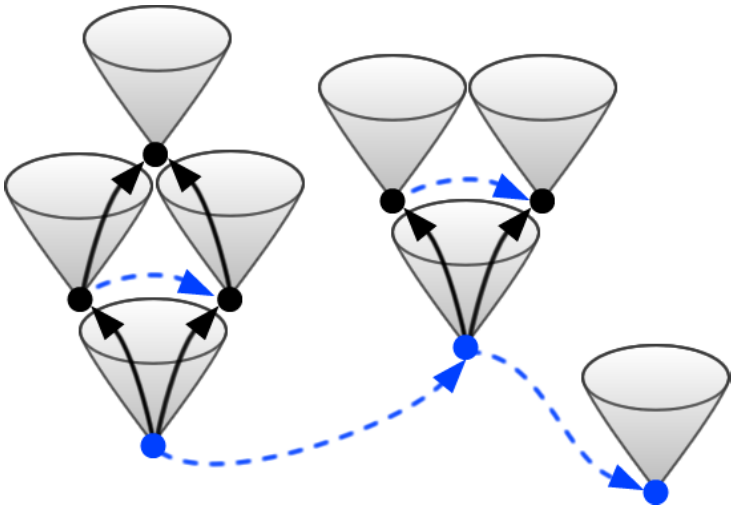


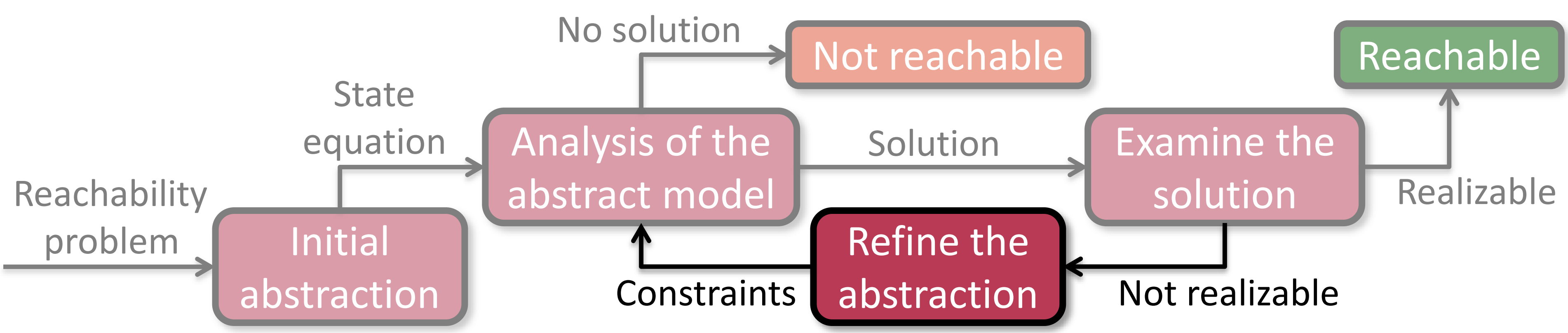


CEGAR approach on Petri nets

$$m_0 + C\mathbf{x} = m_1$$

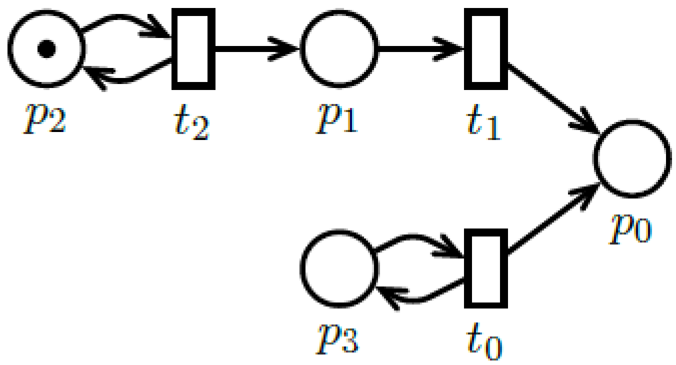
- Traversing the solution space
  - Semi-linear space
    - Base solutions
    - T-invariants
  - Two types of constraints
    - Jump: obtain different base solution
    - Increment: reach non-base solutions by adding T-invariants



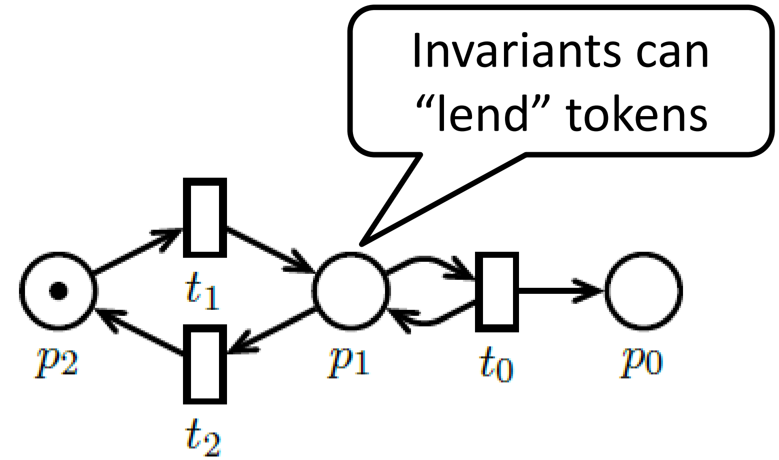


CEGAR approach on Petri nets

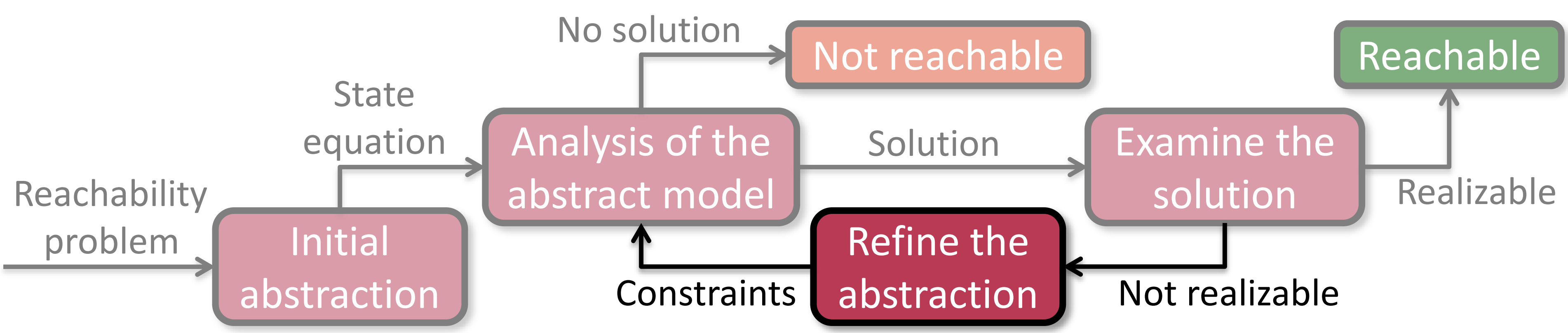
- Constraint examples



$(t_0) \xrightarrow{\text{Jump: } t_0 < 1} (t_2, t_1)$



$(t_0) \xrightarrow{\text{Increment: } t_1 > 0} (t_1, t_0, t_2)$



Our previous results

- Correctness of the algorithm [Hajdu et al.'14]
  - The algorithm may give a wrong answer
  - Detect these cases and also solve some of them
- Completeness of the algorithm [Hajdu et al.'13]
  - The algorithm may…
    - …fail to decide the problem
    - …fail to terminate
  - Improvements, but still incomplete

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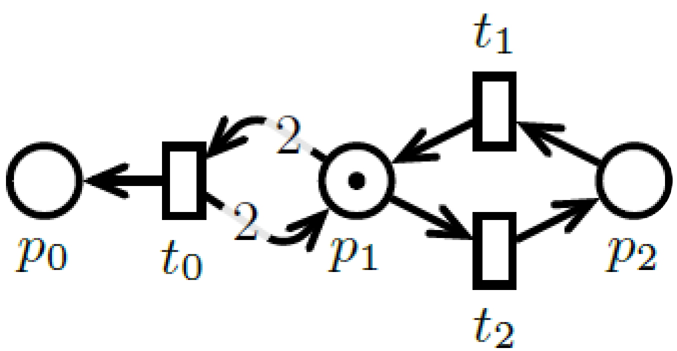
New iteration strategy

- A reason for incompleteness
  - Increment constraints add a T-invariant to a solution
  - Possible cases:
    1. Cannot fire
    2. Fires and enables some other transition
    3. Fires but does not enable any transition
       - Different solution is obtained without any progress → terminate
       - There may be „distant" invariants

} Repeat refinement cycle

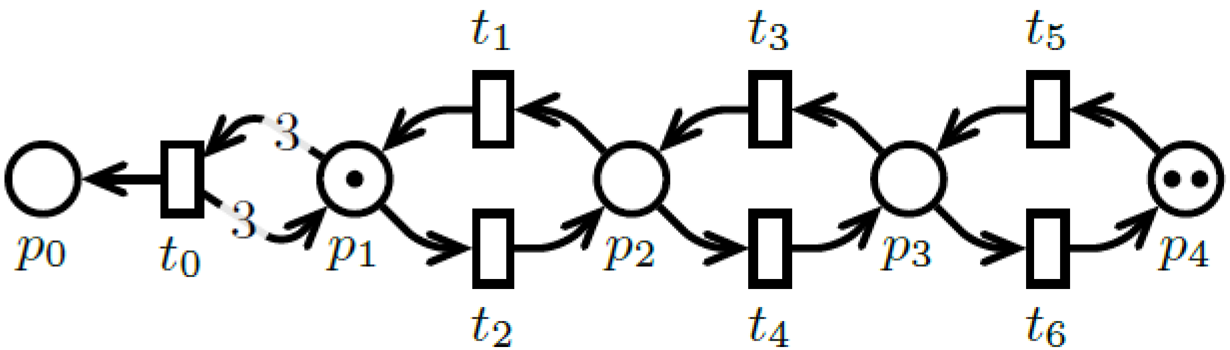


New iteration strategy

- Our new strategy
  - Extending increment constraints
    - “Lending” tokens to places → “lending” tokens to invariants
  - Distant invariant
    - *Z is a distant invariant for Y if Z can produce tokens in places connected to Y*
  - Problems to be solved
    - Number of tokens to “borrow”
    - Termination criterion
      - E.g.: X lends to Y and Y lends to X → infinite loop

New iteration strategy

- Number of tokens „borrowed"
  - One token at a time and repeat
  - Some problems cannot be solved this way
- Termination criterion
  - Form a chain of invariants
    - If Z did not help Y → find distant invariant for (Z + Y)
    - Union of transitions → finite



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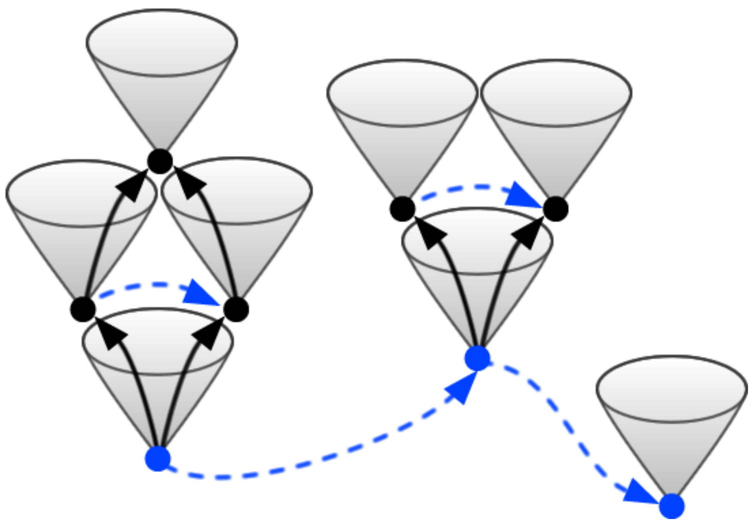
Search strategies

- The algorithm traverses the solution space
  - Multiple jump/increment constraints
  - We examined different strategies
    - Depth-first search
    - Breadth-first search
  - We developed a complex strategy

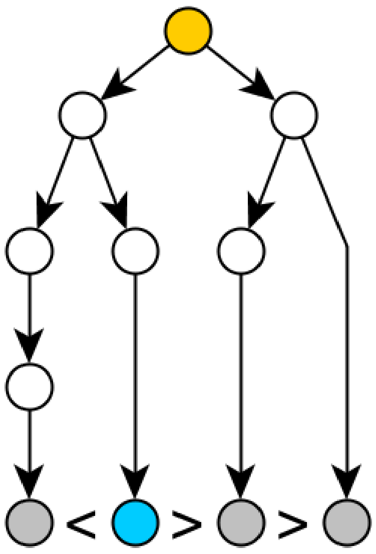


Search strategies

- Depth-first search
  - \+ Efficient regarding memory usage
  - \+ Fast convergence
  - \- May not find the minimal solution
  - \- May not terminate
- Breadth-first search
  - \+ Always finds the minimal solution
  - \- Less efficient than DFS
  - \- May not terminate if there is no solution

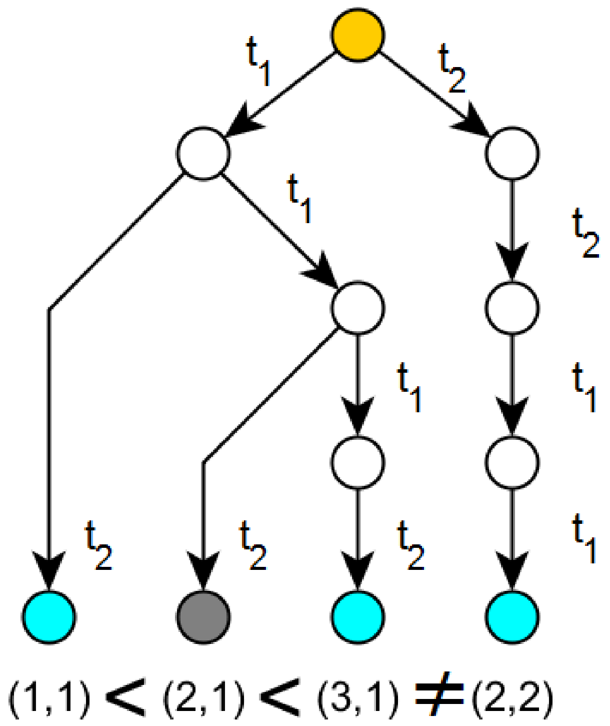
Search strategies

- Complex strategy
  - Based on DFS
  - Expand one level of the solution space
    - All partial solutions of a solution vector
  - Define an ordering between the partial solutions
  - Filter based on the order



Search strategies

- Ordering
  - Partial order: Parikh image of firing sequence
- Filtering
  - Maximal solutions
    - Closest to a realizable solutions
    - Infinite loops can be detected
  - Minimal solutions
    - Slower convergence
    - May involve different T-invariants



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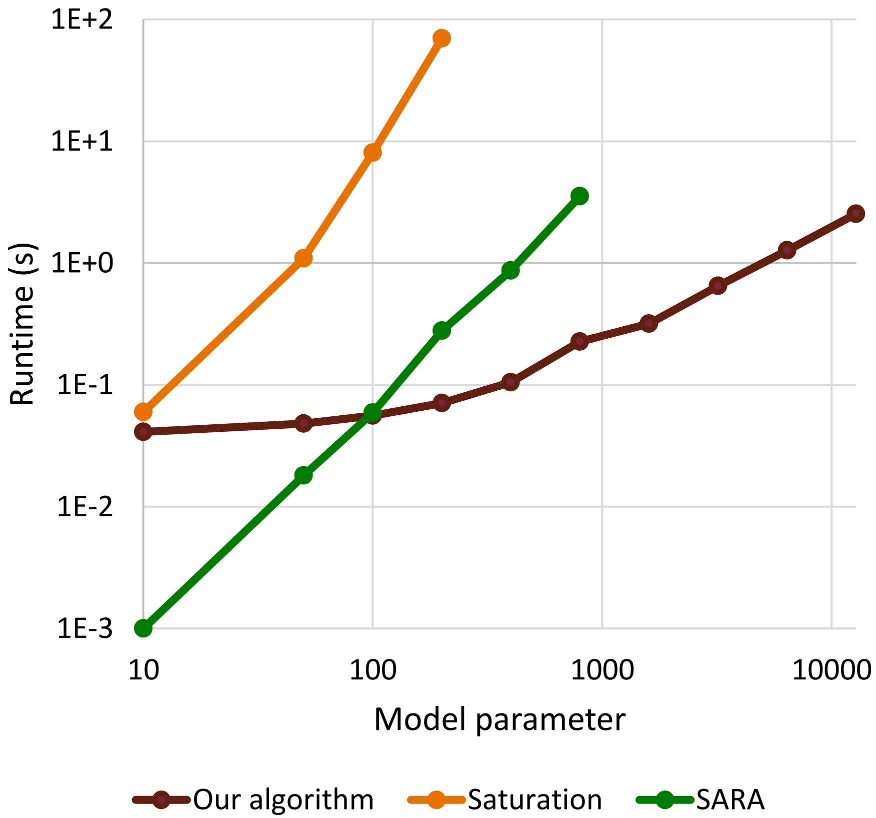
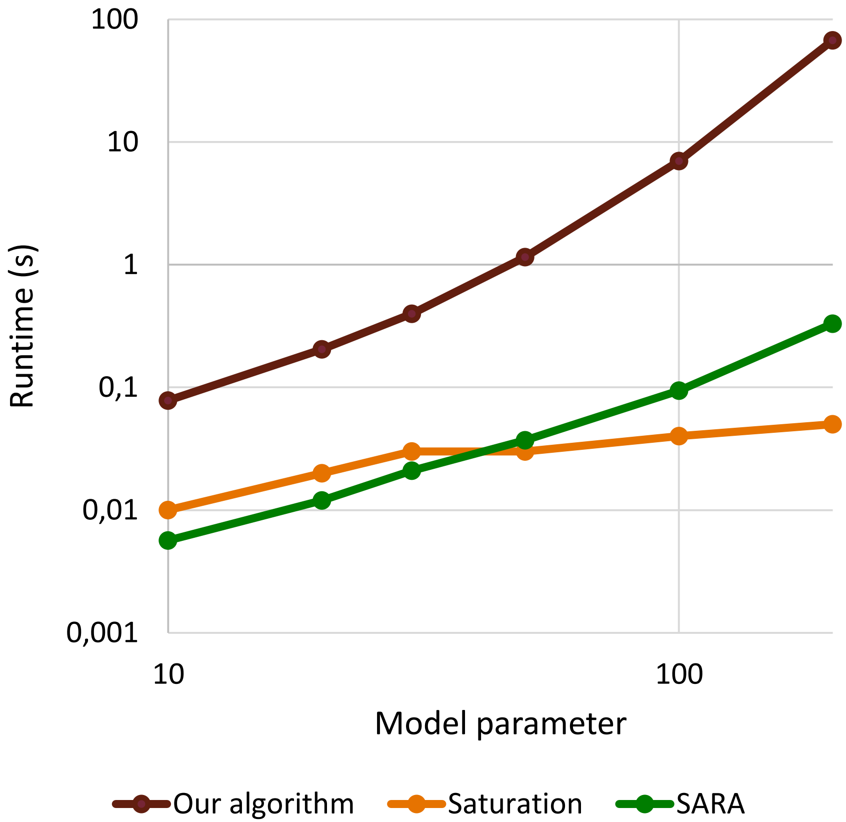
Evaluation

- Implementation: PetriDotNet framework
- Comparison of algorithms
  - SARA tool
    - Wimmel & Wolf
  - Saturation-based method (developed at our group)
    - Symbolic algorithm
- Comparison of search strategies
  - BFS ↔ DFS ↔ Complex strategy

Comparison of the algorithms

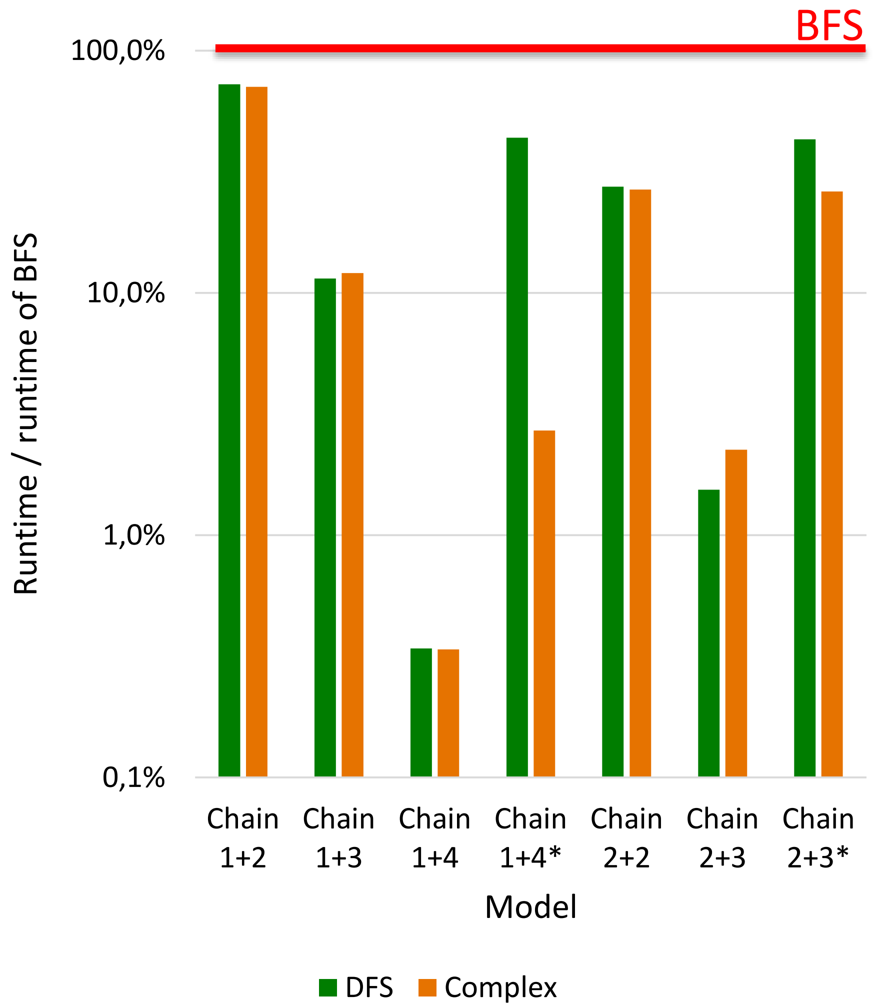
- FMS
  - Flexible manufacturing system
  - Fixed structure
  - Parameter affects state space
- Dining philosophers
  - Illustration of mutual exclusion
  - Structure grows with parameter

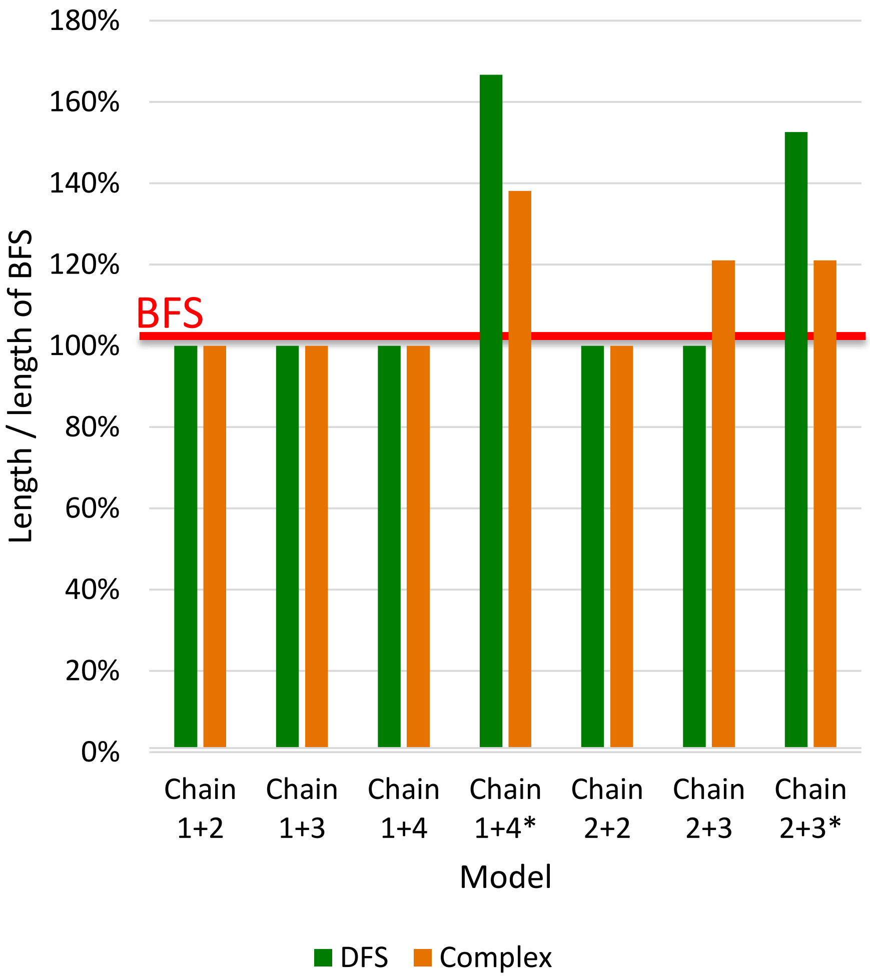




Comparison of search strategies

- Models with large solution space





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Conclusions

- Theoretical results
  - New iteration strategy and limitations
- Practical results
  - Behavior of BFS, DFS and a complex strategy
- Future work
  - Forward reachability: did we reach the limits?
  - How structure and behavior affects performance?

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PetriDotNet: bit.ly/1Rqnare
Measurements: bit.ly/1CoMJSG

References

[Mayr’81] MAYR, EW. "An algorithm for the general Petri net reachability problem." *Proc. 13th Annual ACM Symposium on Theory of Computing, 1981*. 1981.

[Lipton’76] Lipton, Richard. "The reachability problem requires exponential space." *Research Report 62, Department of Computer Science, Yale University, New Haven, Connecticut* (1976).

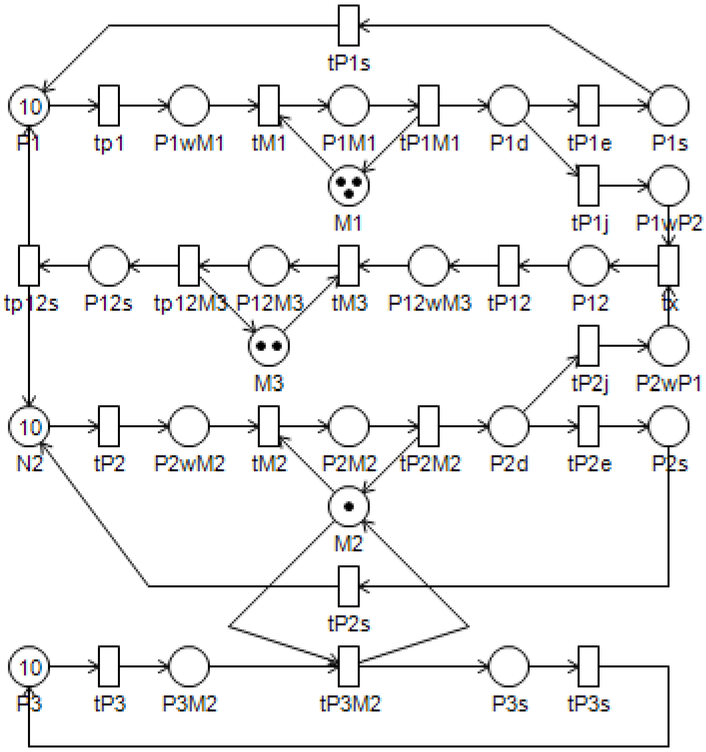
[Wimmel & Wolf’11] Wimmel, Harro, and Karsten Wolf. "Applying CEGAR to the Petri net state equation." *Tools and Algorithms for the Construction and Analysis of Systems*. Springer Berlin Heidelberg, 2011. 224-238.

[Hajdu et al.’13] Hajdu, Ákos, et al. "Extensions to the CEGAR approach on Petri Nets." (2013): 274-288.

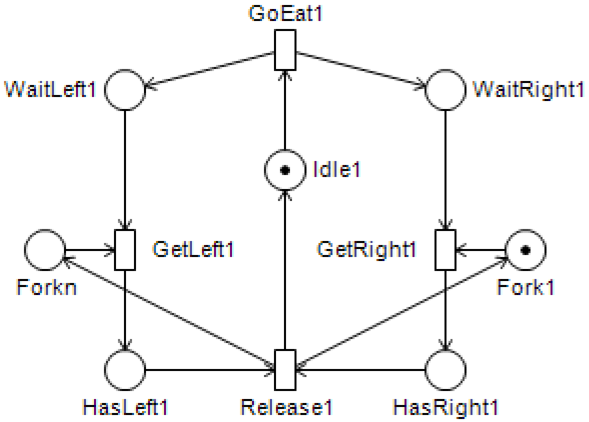
[Hajdu et al.’14] Hajdu, Akos, et al. "Extensions to the CEGAR Approach on Petri Nets." *Acta Cybernetica* 21 (2014): 401-417.

Models

FMS



Dining philosophers



Chain 2+3